Density Function:

$P(x)$ or $F(x)$

① $0 \leq F(x) \leq 1$

② $\int_{-\infty}^{\infty} F(x)\,dx = 1$ for continous

$\sum_{-\infty}^{\infty} F(x) = 1$ for Discreate

Commulative Distribution Function $F(x)$

$F(x) = P(X \leq x)$

$F(3) = P(3) + P(2) + P(1)$ For Discreate

① $F(-\infty) = 0$

② $F(\infty) = 1$

③ $F(a) < F(b)$ if $a < b$

④ increasing function

$$f(x) = \frac{dF(x)}{dx}$$

$$F(x) = \int_{-\infty}^{x} f(x)\,dx$$

Sheet 4

[1] $f(x) = \begin{cases} x & 0 \leq x \leq 1 \\ \frac{3-x}{4} & 1 \leq x \leq 3 \\ 0 & \text{otherwise} \end{cases}$

Prove that $f(x)$ is dinisty function then Find $F(x)$

$$\int_{-\infty}^{0} 0\,dx + \int_{0}^{1} x\,dx + \int_{1}^{3} \frac{3-x}{4}\,dx + \int_{3}^{\infty} 0\,dx$$

$$= \int_{-\infty}^{\infty} f(x)\,dx = \frac{x^2}{2}\Big|_0^1 + \left(\frac{3x}{4} - \frac{x^2}{8}\right)_1^3$$

$$= \frac{1}{2} + \left(\frac{9}{4} - \frac{9}{8}\right) - \left(\frac{3}{4} - \frac{1}{8}\right)$$

$$= 1$$

$\therefore$ $f(x)$ is dinisty function

$$F(x) = \int_{-\infty}^{x} f(x)\,dx$$

For $-\infty \to 0$

$F(x) = F(-\infty) + F(0) = 0$

For $0 \to 1$

$$F(x) = F(0) + \int_0^x x\,dx = 0 + \frac{x^2}{2}$$

$$= \frac{x^2}{2}$$

For $1 \to 3$

$$F(x) = F(1) + \int_1^x \left[\frac{3}{4} - \frac{x}{4}\right] dx$$

$$= \frac{(1)^2}{2} + \left[\frac{3x}{4} - \frac{x^2}{8}\right]_1^x$$

$$= \frac{1}{2} + \left(\frac{3x}{4} - \frac{x^2}{8}\right) - \left(\frac{3}{4} - \frac{1}{8}\right)$$

$$= \frac{3x}{4} - \frac{x^2}{8} - \frac{1}{8}$$

For $3 \to \infty$

$$F(x) = F(3) + \int_3^x 0\,dx$$

$$= \frac{3(3)}{4} - \frac{(3)^2}{8} - \frac{1}{8}$$

$$= 1$$

$$F(x) = \begin{cases} 0 & -\infty \leq x \leq 0 \\ \frac{x^2}{2} & 0 \leq x \leq 1 \\ \frac{3x}{4} - \frac{x^2}{8} - \frac{1}{8} & 1 \leq x \leq 3 \\ 1 & 3 \leq x \leq \infty \end{cases}$$

[2] if $f(x) = \frac{1}{2^x}$ for $x = 1, 2, 3, \ldots$

Can $f(x) \to$ Probability function

$$\sum_{-\infty}^{\infty} f(x) = \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \cdots$$

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$$\sum_{-\infty}^{\infty} f(x) = \frac{r}{1-a}$$

$r = \frac{1}{2}$ $\quad a = \frac{1}{2}$

$$\sum_{-\infty}^{\infty} f(x) = \frac{\frac{1}{2}}{1-\frac{1}{2}} = \frac{\frac{1}{2}}{\frac{1}{2}} = 1$$

$\therefore$ $f(x)$ is a dinisty function

$\boxed{4}$ IF .

$$f(x) = \begin{cases} k(2-x) & 0 \le x \le 2 \\ 0 & \text{other wise} \end{cases}$$

Find k for f(x) to be Dinisty Function

$$\int_0^2 k(2-x)dx = 1 = \int_{-\infty}^{\infty} f(x)\,dx$$

$$2kx + \frac{kx^2}{2}\Big|_0^2 = 1$$

$$4k - 2k = 1$$

$$k = 0,5$$